

DECISION ANALYSIS NEWSLETTER

Published by the ORSA Special Interest Group on Decision Analysis

VOLUME 9, NUMBER 2

August, 1990

Editor's Note

Just a reminder that we are eager to publish abstracts of all papers in the area of Decision Analysis, broadly conceived. The only requirements for our publishing an abstract of your work are:

1) That the paper itself not have appeared in print yet; 2) that it is available for distribution upon request; and 3) that the abstract not exceed 200 words by much.

If there is a charge, please so indicate when you send your *complete paper* to the editor:

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Please phone or write in any changes in your activities or employment that could be of interest to our membership.

Please Note: (1) Inform the ORSA business office at Mount Royal and Guilford Avenues, Baltimore, MD 21202 of address change; we get our mailing labels from them! Thanks!

(2) To be included on the mailing list, you should join the Special Interest Group on Decision Analysis: send letter to ORSA office and \$3 (\$5) for an ORSA (non)member.

Frank P. Ramsey Medal

In 1985, the ORSA Special Interest Group on Decision Analysis established the Frank P. Ramsey Medal to honor people who have made distinguished contributions to the field of Decision Analysis. In addition to the medal, the recipient receives a check for \$1,000, generously provided by funds from Applied Decision Analysis, Inc.; Decision Focus, Inc.; and Strategic Decisions Group; along with SIG funds. At the Las Vegas meeting in May, the SIG announced the 1989 and 1990 winners, the winners having been allocated to years randomly. Their identities are revealed by the citations appearing on page two. Their acceptance statements appear on page 3. In inverse order, the 1990 Medalist will present an extended address at the Philadelphia TIMS/ORSA meeting, while the 1989 Medalist will do the same at Nashville next Spring.

Allen Miller at ADA

Dr. Allen C. Miller has joined Applied Decision Analysis, Inc. as a Senior Scientist. The firm's address is 3000 Sand Hill Road, Menlo Park, CA 94025; phone (415) 854-7101.

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1989 Ramsey Medal Citation

for

Ralph L. Keeney

Ralph L. Keeney has significantly advanced both the state of practice and the theoretical foundations of multiobjective decision analysis. He has written about and participated in a large body of applied work that serves as a guide for others to follow. His work in the early seventies provided the theoretical basis of defensible elicitation of multiattribute utility functions for decision making under uncertainty. This work and associated case examples culminated in the 1976 book with Howard Raiffa, Decisions with Multiple Objectives, which won ORSA's Lanchester Prize. This work still serves as a primary reference on this topic.

Through all of his work, Ralph has been a consistent and articulate voice for the fundamental role of values in decision making and for the importance of quantitative analysis in complex decisions, particularly for public policy issues. There are clear signs that this message is leading to changes in the manner in which decision making is done in important public arenas. This medal is offered in gratitude for all of Ralph's contributions to the profession.

1990 Ramsey Medal Citation

for

Robert L. Winkler

Robert L. Winkler is well known for his work on subjective probability assessment, probabilistic forecasting, evaluation of probabilistic forecasts, and Bayesian consensus models. His long list of publications includes as well significant work on Bayesian statistical modeling, group decision making, risk assessment, utility theory, Bayesian financial models, time series analysis, and other topics.

Bob has contributed to the training of many scholars in decision analysis, and to the development of decision analysis centers at Universities. He has developed outstanding introductory textbooks and been a leader in the teaching of decision analysis.

Considerable editorial responsibility and service to professional groups have been a part of Bob's career from the beginning. Bob has contributed greatly to the intellectual base for decision analysis and in the dissemination of these ideas to others. No doubt, he will continue to do so for many years to come. With these thoughts in mind, it is an honor to offer him the Ramsey medal.

Winkler at INSEAD --- 1990 Ramsey Medalist Bob Winkler is at INSEAD for this academic year, except for a trip back to participate in the Philadelphia meeting this October. His address, phone numbers, e-mail, and FAX are as follows:

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Ramsey Medal Acceptance

by

Robert L. Winkler

I'm very honored to receive the Ramsey Medal, and would like to thank the Special Interest Group, the Council, and the many professional colleagues who have provided inspiration and guidance over the years. The list of previous winners speaks for itself, and it is an honor to follow this distinguished group. I will try not to tarnish the medal too much. I am especially pleased to receive this award simultaneously with Ralph Keeney. I have a great deal of respect and admiration for Ralph's work, and we have been friends and sometimes collaborators for about two decades.

One of the things that makes decision analysis an exciting field is the limitless variety of important decision-making problems that are potential applications. As Ralph pointed out, the problems facing society are not getting easier, and this is an important challenge for those interested in decision analysis. Another appealing feature about decision analysis is the interdisciplinary nature of the field. A quick perusal of the list of Ramsy Medal winners, with their different backgrounds and orientations, illustrates this. We need to take advantage of this interdisciplinary background and to utilize insights from different areas creatively as we attempt to improve both the theory and practice of decision analysis.

In closing, I would just like to once again express my gratitude for this tremendous honor.

Ramsey Medal Acceptance

by

Ralph L. Keeney

I would like to thank the Special Interest Group and the Council for this honor. It is a special honor for two reasons: recognition by peers is the highest type of professional recognition and I have tremendous professional respect for all of the previous Ramsey Medal recipients, Howard Raiffa, Ron Howard, Peter Fishburn, and Ward Edwards. It is also a pleasure to receive this award at the same time as Bob Winkler, as I have had the privilege of working with Bob on many occasions and have been his colleague and friend for several years.

I'd like to take this opportunity to make two important points. First, decision analysis provides us with rather unique tools to make important contributions to important problems. And second, wisely applying these tools can make a significant difference on many problems.

The problems facing society are not getting easier. The skills of decision makers are not increasing at a rate greater than the growth in problem complexity. And the usefulness of decision analysis is not limited by its philosophical foundations, theory, or practice. If we are limited, it's by our willingness to dig deeply into real problems, to structure them, to be creative in developing alternatives and analyzing them, and to communicate the insights. Problems such as global warming, disarmament, the crisis with savings and loan institutions, overpopulation, illegal aliens, AIDS, drug use, clean air, and global competition, each offer thousands of decision problems where quality decision analysis can make a difference. My hope is that we collectively recognize this as a challenge, accept it, and make these differences.

PAPERS RECEIVED

Please request copies directly from the author, not the Newsletter Editor

From **Richard O. Beil, Jr.**, and **T. Randolph Beard**, Dept. of Economics, Auburn University, Auburn, Alabama 36849-5242:

Do People Rely on the Utility Maximization of Others?: An Experimental Test.

The assumption of utility maximization by agents forms one of the cornerstones of modern economic theory. This maximization assumption, however, is quite different from the notion that agents rely on the utility maximizing behavior of others. This distinction is important in many game theoretic settings, and is central to the issues that arise in "refining" the Nash equilibrium concept.

In this paper we evaluate the degree to which economic agents rely on the utility maximizing behavior of others using experimental techniques. We find that while agents' behavior is nearly always consistent with maximization, a statistically significant proportion of agents do not rely on the maximizing behavior of their rivals. The experimental results are evaluated from several perspectives, and the implication of our work for the Nash refinement problem is discussed.

From **Harold P. Benson**, College of business Administration, University of Florida, Gainesville, FL 32611, and **Yasemin Aksoy**, A. B. Freeman School of Business, Tulane University, New Orleans, LA 70118:

Using Efficient Feasible Directions in Interactive Multiple Objective Linear Programming.

In this paper we show how to incorporate efficient feasible directions into interactive line search algorithms for multiple objective linear programming problems. The resulting new line search is guaranteed to generate points that lie in the efficient set. In addition, the new line search automatically corrects possible errors in judgment that the decision maker may inadvertently commit in giving some of his responses. These two improvements are achieved without sacrificing the key property of typical interactive line search algorithms, namely that they yield a new point at each iteration more preferred by the decision maker than the current point. The only additional computational requirement of the new line search is the solution of a single linear programming problem at each iteration. For these reasons, we advocate the incorporation of efficient feasible directions into interactive line search algorithms for multiple objective linear programs in the manner shown in this paper.

From **Patrick L. Brockett**, Dept. of Finance, Graduate School of Business, The University of Texas, Austin, TX 78712, and **Yehuda Kahane**, Faculty of Management, Tel Aviv University, Tel Aviv, Israel:

Risk, Return, Skewness, and Preference.

This paper considers choice between individual projects and shows that when the choice set includes arbitrary distributions, then *any* assumed relationship between utility theory and general moment preferences for individual decision makers is theoretically unsound. In particular, a risk averse investor with *any* common utility function may, when

choosing between two positive return opportunities, prefer the project simultaneously having a lower mean, higher variance, and lower positive skewness. Moreover the decision maker can prefer opportunities with higher variance even when the opportunities are continuous, unimodal, and arbitrarily visually and statistically close to the normal distribution in shape. Our conclusions hold for any decision maker with a utility function whose derivatives alternate in sign (i.e., we exclude the uninteresting cases of quadratic and cubic utilities). The method of analysis is based upon the theory of Tchebychev systems of functions which deals with the expected value of [utility] functions of stochastic variables with known moments, and can be generalized to include any number of moments and any order of moment preference. The paper significantly extends the current stochastic dominance literature by generalizing and clarifying its results. Although we focus on the first three moments, the results, as presented here, apply to all higher moments as well. It is also shown that there can be extremely large deviations between the certainty equivalents of distributions having the same moments, so this result is also pertinent to practical decision analysis as well.

From **Richard L. Daniels**, Fuqua School of Business, Duke University, Durham, NC 27706, and **L. Robin Keller**, Decision, Risk, and Management Science Program, National Science Foundation, Room 336, 1800 G Street, N.W., Washington, DC 20550:

Choice-Based Assessment of Utility Functions.

An approach for calibrating utility functions in which consistent paired-comparison response modes are used for both elicitation and descriptive validation is proposed and experimentally evaluated. The choice-based procedure presented has the potential to avoid the systematic biases in functions assessed via indifference judgments such as probability or certainty equivalents. Our results indicate that the choice-based assessment procedure outperforms calibration approaches based on indifference judgments in predicting subjects' choices among risky options for the two types of utility models studied, von Neumann-Morgenstern expected utility and lottery dependent expected utility.

From **Peter C. Fishburn**, Room 2C-354, AT&T Bell Laboratories, 600 Mountain Ave., Murray Hill, NJ 07974:

Additive Differences and Simple Preference Comparisons.

Two axiomatizations of the multiattribute additive difference representation of preferences are presented. One uses topological assumptions to obtain a continuous representation; the other uses a more general algebraic structure. Both use only simple preference comparisons and require three or more attributes. Proofs of the representation theorems are based on recent work in nontransitive additive conjoint measurement. Specializations of the additive difference model for homogeneous product sets are also axiomatized. Contexts for these specializations include time streams and finite-states decision under uncertainty.

Additive Nontransitive Preferences.

This note summarizes recent contributions to multiple-factor nontransitive preference theory. It considers representations in which preferences are reflected by positive sums of intrafactor utilities.

Continuous Nontransitive Additive Conjoint Measurement.

This paper axiomatizes a nontransitive generalization of additively separable utility for preferences on multiattribute outcomes. The axiomatization requires at least three factors or attributes, retains the essential independence aspect of additive conjoint measurement, and makes no assumption about transitive preferences. The real valued functions on the factors in the numerical representation are continuous and unique up to similar proportionality transformations. Comparisons are made to related work by Karl Vind.

Nontransitive Preferences in Decision Theory.

Intransitive preferences have been a topic of curiosity, study and debate over the past 40 years. Many economists and decision theorists insist on transitivity as the cornerstone of rational choice, and even in behavioral decision theory intransitivities are often attributed to faulty experiments, random or sloppy choices, poor judgment, or unexamined biases. But others see intransitive preferences as potential truths of reasoned comparisons and propose representations of preferences that accommodate intransitivities.

This paper offers a partial survey of models for intransitive preferences in a variety of decisional contexts. These include economic consumer theory, multiattribute utility theory, game theory, preference between time streams, and decision making under risk and uncertainty. The survey is preceded by a discussion of issues that bear on the relevance and reasonableness of intransitivity.

Nontransitive Preferences and Normative Decision Theory.

Nontransitive preferences arise naturally in several contexts for decision making and deserve recognition in normative decision theory. Numerical utility models for nontransitive preferences are described.

On Nonstandard Nontransitive Additive Utility.

A transparent finite cancellation condition is shown to be necessary and sufficient for a nonstandard additively separable utility representation of a preference relation on an arbitrary subset of a finite-dimensional product set. The representation makes no presumptions concerning transitivity, completeness or continuity.

On The Theory of Ambiguity.

This paper considers axioms and numerical representations for event ambiguity as a primitive concept without direct reference to likelihood, subjective probability, preference or decision.

From **Peter C. Fishburn**, Rm 2C-354, AT&T Bell Laboratories, 600 Mountain Ave., Murray Hill, NJ 07974, and **D. Marc Kilgour**, Dept. of Mathematics, Wilfrid Laurier University, Waterloo, Ontario, Canada N2L 3C5:

Binary 2×2 Games.

The 2×2 game is the simplest interactive decision model that portrays concerned decision makers with genuine choices. There are two players, each of whom must choose one of two strategies, so that there are four possible outcomes. *Binary 2×2 games* are 2×2

games with no restrictions on the players' preference relations over the outcomes. They therefore generalize the strict ordinal 2×2 games and the ordinal 2×2 games, classes which have already been studied extensively. This paper enumerates the strategically distinct binary 2×2 games. It also identifies important subsets defined by the number of pure Nash equilibria and the occurrence of dominant strategies.

From **Peter C. Fishburn**, Rm 2C-354, AT&T Bell Laboratories, 600 Mountain Ave., Murray Hill, NJ 07974, and **Irving H. LaValle**, A. B. Freeman School of Business, Tulane University, New Orleans, LA 70118:

Nonstandard Nontransitive Utility on Mixture Sets.

We axiomatize three increasingly restrictive representations for a preference relation on a mixture set that presume neither transitivity nor an Archimedean condition. The representations involve a function from ordered pairs of elements in the mixture set into an ordered field extension of the reals. The axioms are formulated as finite cancellation conditions and are necessary and sufficient for the representations. The least general representation is a nonstandard extension of SSB (skew symmetric bilinear) utility.

From **Peter C. Fishburn**, Rm 2C-354, AT&T Bell Laboratories, 600 Mountain Ave., Murray Hill, NJ 07974, and **Rakesh K. Sarin**, Fuqua School of Business, Duke University, Durham, NC 27706:

Dispersive Equity and Social Risk.

Dispersive equity is concerned with the impact of life-threatening risks from alternative policy decisions on homogeneous groups in a population. It is not addressed to the disutility of various numbers of fatalities that might occur, but rather to how fatalities are distributed over the groups. Although dispersive equity has long been recognized as an important component of some policy decisions, it has received little formal treatment within the matrix of salient factors for such decisions. Our purpose is to advance its formal development.

This is done within a formulation based on probability distributions over potential fatality sets. The formulation gives rise to four distinct but not independent equity concepts: individual risk equity, group risk equity, dispersive equity, and social outcome equity. Each of these is analyzed, but special attention is given to dispersive equity. The paper also comments on relationships between total equity and its four components, and between total equity and the disutility of various numbers of fatalities.

From **Gordon B. Hazen**, IE&MS Dept., McCormick School of Engineering and Applied Science, Northwestern University, Evanston, IL 60201-9990:

Decision Versus Policy: An Expected Utility Resolution of the Ellsberg Paradox.

An expected utility maximizer who wishes to establish a choice *policy* (rather than merely make a single choice) in an Ellsberg urn scenario will have reason to distinguish between urns whose contents are more or less ambiguous. Specifically, risks averters will avoid ambiguity and risk seekers will prefer ambiguity. A resolution of Ellsberg's paradox is thereby provided, in which ambiguity aversion/seeking is "rational", but the normative status of expected utility remains unassailed.

From **Jean-Yves Jaffray**, Laboratoire d'Informatique de la Décision, Université de Paris VI, 4 Place Jussieu, 75005-Paris, France:

Bayesian Updating and Belief Functions.

In a wide class of situations of uncertainty, the available information concerning the event space can be described as follows: There exists a true probability which is only known to belong to a certain set \mathcal{P} of probabilities; moreover, the lower envelope f of \mathcal{P} is a belief function, i.e., a ∞ -monotone capacity, and characterizes \mathcal{P} , i.e., \mathcal{P} is the set of all probabilities which dominate f . This is in particular the case when data result from large-scale sampling with incomplete observations.

This study is concerned with the effect of conditioning on such situations. The natural conditioning rule is here the Bayesian rule: there exists a posterior probability after the observation of event E , and it is known to be located in \mathcal{P}^E , set of the conditionals of the members of \mathcal{P} .

The lower envelope f^E of \mathcal{P}^E is shown to be itself a belief function and the explicit expression of the Möbius inverse, ϕ^E , of f^E in terms of that, ϕ , of f is exhibited. However, f^E no longer characterizes \mathcal{P}^E (not all probabilities dominating f^E belong to it) unless stringent NS conditions are satisfied by f .

The difficulties resulting from this fact are discussed and suggestions to cope with them are made.

From **L. Robin Keller**, Decision, Risk, and Management Science Program, National Science Foundation, Room 336, 1800 G Street, N.W., Washington, DC 20550:

Properties of Utility Theories and Related Empirical Phenomena.

This paper provides an overview of some key properties of expected utility theory and a discussion of related empirical phenomena. The link between these properties and phenomena and developments in generalized utility theories is also discussed.

From **Craig W. Kirkwood**, Dept. of Decision and Information Systems, Arizona State University, Tempe, AZ 85287-4206:

A Tutorial Summary of Methods for Applied Decision Analysis.

This tutorial presents useful applied decision analysis methods that are often not covered in introductory textbooks. It discusses situations where decision analysis is particularly useful, reviews basic concepts, and presents methods for dealing with analysis complexity. These methods include influence diagrams, algebraic approaches, and approximations for probability distributions and utility functions. Issues in subjective probability elicitation are also reviewed, as well as tools for analyzing decisions with multiple objectives. Solution procedures for decision models are considered, and source code in Pascal is included for a decision model solver.

From **Kathryn Blackmond Laskey**, Decision Science Consortium, Inc., 1895 Preston White Drive, Reston, VA 22091:

A General Class of Conjugate Priors for Analysis of Variance Models.

A class of models for the prior distribution of the mean for n -way AOV models is presented. The class generalizes the model suggested by Lindley and Smith for two-factor designs. The models are special cases of the normal/inverted chi-square natural conjugate prior. The number of hyperparameters required to specify the model varies depending on the stringency of exchangeability assumptions. Assessment of hyperparameters is discussed.

From **Irving H. LaValle**, A. B. Freeman School of Business, Tulane University, New Orleans, LA 70118, and **Peter C. Fishburn**, Rm 2C-354, AT&T Bell Laboratories, 600 Mountain Ave., Murray Hill, NJ 07974:

Lexicographic State-Dependent Subjective Expected Utility.

An additive-across-states decomposition of lexicographic linear utility obtains under a mild structural assumption concerning sufficient richness of acts in the domain of preference assessment, but the vectorial nature of lexicographic utility introduces two complexities absent in the real-valued case. First, the concept of state nullity becomes lexicographic rather than binary; and second, a standard construction for obtaining subjective probabilities from real-valued, state-dependent utilities produces matrices instead of nonnegative real numbers in the lexicographic setting.

Linear Lexicographic State-Dependent Utility.

If preference is a fully linear but not necessarily continuous weak order, a state-dependent, additive decomposition of linear lexicographic utility obtains under the same structural assumption that suffices in the continuous, real-valued-utility case. Some ramifications of states-additive, linear lexicographic utility representations are outlined in this paper.

From **David Matheson**, 376 College, #5, Palo Alto, CA 94306:

When Should You Reexamine Your Frame?

Descriptive studies have shown that in solving a problem, the frame--what constitutes the problem--shifts at various times. Some of these times seem hard to justify from a normative perspective. Therefore, a normative theory of when frames should shift is needed.

This paper provides the beginning of such a theory. It codifies the insights in the literature on problem-solving processes into the *frame-shift model* and refines the model enough so that specific frame management questions can be asked and modelled. One approach to answering the question of "when to reexamine the frame?" is to establish a *working limit*. A problem-solving team works within the problem as framed until it finds a solution, it perceives a challenge to the frame, or until it reaches the working limit. When it reaches the working limit, it shifts attention to reexamining the frame.

The *working-limit model* is a decision analytic scheme for setting this working limit. In certain situations, the optimality conditions reflect intuitive notions of justifying continued work on the basis of progress. A well-founded decision rule is to work as long as a

probabilistic *achievement rate* is larger than the *depreciation rate* of value. In some cases, it can be shown that the working limit decision is unimportant: the team should either never start working within a frame, or never reexamine it unless it is challenged.

From **Mark J. Machina**, Dept. of Economics, University of California-San Diego, La Jolla, CA 92093, and **David Schmeidler**, Dept. of Statistics, Tel-Aviv University, 69978 Tel-Aviv, Israel:

A More Robust Definition of Subjective Probability.

Although their goal is to separate a decision maker's underlying *beliefs* (their subjective probabilities of events) from their *preferences* (their attitudes toward risk), classic choice-theoretic derivations of subjective probability all rely upon some form of the Marschak-Samuelson "Independence Axiom" or the Savage "Sure-Thing Principle," which is equivalent to requiring that the decision maker's preferences over lotteries conform to the expected utility hypothesis. This paper presents a choice-theoretic derivation of subjective probability which satisfies the axioms of classical probability theory, but which neither assumes nor implies that the decision maker's preferences over lotteries necessarily conform to the expected utility hypothesis.

From **Robert F. Nau**, Fuqua School of Business, Duke University, Durham, NC 27706:

Indeterminate Probabilities and Utilities on Finite Sets.

This paper presents a quasi-Bayesian model of indeterminate probabilities on finite state spaces in which indeterminacy is attributed to measurement-induced perturbations in beliefs: an "uncertainty principle" for psychological observations. The model is derived from a system of axioms of binary preferences which differs from standard axiom systems insofar as completeness is not assumed and transitivity is weakened. Under this system, beliefs are represented by lower and upper probabilities qualified by numerical confidence weights which can be operationally elicited through the acceptance of bets with limited stakes, a generalization of the operational method of de Finetti. Assessments of confidence-weighted probabilities are dually characterized by concave functions on sets of probabilities, which behave like fuzzy-set membership functions. Hence, the model may be considered to operationalize the idea of "fuzzy" subjective probabilities. Inferences from such assessments can be alternatively summarized by distributions over lower and upper probabilities; hence, the model also operationalizes the idea of "type II" probability distributions. This representation of beliefs is shown to lend itself to the aggregation of judgments elicited from different individuals, circumventing some of the difficulties encountered in aggregating assessments of ordinary (point-valued) probabilities. The device of extraneous scaling probabilities (horse lotteries) can be used to define confidence-weighted lower and upper expected utilities for lotteries on finite sets of states and consequences, a generalization of the subjective expected utility model of Anscombe and Aumann.

From **Clemens Puppe**, Institut für Statistik and Mathematische Wirtschaftstheorie, Universität Karlsruhe, Postfach 6980, D-7500 Karlsruhe 1, West Germany:

The Irrelevance Axiom, Relative Utility and Choice Under Risk.

Transitive preferences over probability measures that satisfy the so-called irrelevance axiom are completely determined by the evaluation of elementary lotteries with only two possible outcomes one of which is fixed. On the other hand, the irrelevance principle does not imply any specific form of the representation on the set of elementary lotteries. In this

paper a relative utility index is introduced to axiomatize preferences that admit a representation which is homogeneous in the probabilities on the set of elementary lotteries. As a further application of the relative utility index new characterizations of expected utility theory, anticipated utility theory and the dual theory of choice under risk are given.

Reduction of Decision Problems: Some Remarks on Rank-Dependent Utility Theory.

Rank-dependent utility theory is a model of decision making under risk where the representation of preferences over probability distributions is based on a (generalized) utility function defined on the outcome/probability-plane. A distinguishing feature of this model is provided by the fact that in rank-dependent utility theory decision problems are reduced to the evaluation of elementary lotteries with only two outcomes one of which is fixed. On the set of elementary lotteries various specific behavioral conditions and their influence on the shape of the generalized utility function can be examined by means of a relative utility index. This method allows one to characterize anticipated utility theory, expected utility theory and the dual theory of choice which are all special cases of the rank-dependent utility approach. It turns out that these theories are in some respect very restrictive. Therefore a more general model within the framework of rank-dependent utility theory is suggested.

From **Peter P. Wakker**, Fuqua School of Business, Duke University, Durham, NC 27706; After 10/1/90--Psychological Lab, University of Nijmegen (NICI), P. O. Box 9104, 6500 HE Nijmegen, The Netherlands:

Additive Representations of Preferences on Rank-Ordered Subsets of Cartesian Products; Part I: The Algebraic Approach.

This paper considers additive conjoint measurement on subsets of Cartesian products containing 'rank-ordered' n -tuples. Contrary to what has often been thought, additive conjoint measurement on subsets of Cartesian products has characteristics different from additive conjoint measurement on full Cartesian products.

Additive Representations of Preferences on Rank-Ordered Subsets of Cartesian Products; Part II: The Topological Approach.

Additive representation theory on subsets of Cartesian products has characteristics different from additive representation theory on full Cartesian products. This paper shows the difficulties that may arise on subsets. These difficulties have been underestimated in the literature. For the special case of rank-ordered subsets of Cartesian products this paper obtains additive representation results. These find application in the modern rank-dependent approaches to decision making under risk/uncertainty, and to generalizations of the Gini index in the measurement of inequality.

A Behavioral Foundation for Fuzzy Measures.

In Savage (1954) a 'behavioral foundation' was given for subjective probabilities, to be used in the maximization of expected utility. This paper analogously gives a behavioral foundation for fuzzy measures, to be used in the maximization of the 'Choquet-integral'. This opens the way to empirical verification or falsification of fuzzy measures, and frees them of their 'ad hoc' character. A self-contained elementary proof is given of Schmeidler's (1989, *Econometrica*) result.

Additive Representations of Preferences, A New Foundation of Decision Analysis; The Algebraic Approach.

In Wakker (1989b, 'Additive Representations of Preference, A New Foundation of Decision Analysis'), a new foundations of decision analysis was given. the main tool was a way to derive comparisons of 'tradeoffs' from ordinal preferences, with comparisons of tradeoffs revealing orderings of utility differences. These comparisons of tradeoffs underly the construction of standard sequences in conjoint measurement theory. The restrictive structural assumption (every approach has its restrictive structural assumption) was of a topological nature, requiring continuity. This paper adapts the main results of Wakker (1989b) to the algebraic approach, where a solvability condition is required which is less restrictive than continuity.

From Finite - To Infinite - Dimensional Integral Representations; Unbounded Utility for Savage (1954) and Others.

A general procedure for extending finite-dimensional 'additive-like' representations to infinite-dimensional 'integral-like' representations is developed by means of a condition called 'truncation-continuity'. The restriction of boundedness of utility, met throughout the literature, can now be dispensed with, and for instance normal distributions, or any other distribution with finite first moment, can be incorporated. Classical representation results of expected utility, such as Savage (1954), von Neumann & Morgenstern (1944), Anscombe & Aumann (1963), de Finetti (1937), and many others, can now be extended. The results are generalized to Schmeidler's approach with nonadditive measures and Choquet integrals. The different approaches have been brought together in this long paper to bring to the fore the unity in the extension process.

Appeared Recently

- P. L. Yu and D. Zhang, "Competence Set Analysis for Effective Decision Making", *Control-Theory and Advanced Technology*, 5 (1989), 523ff.
- I. S. Chien, P. L. Yu, and D. Zhang, "Indefinite Preference Structures and Decision Analysis", *Journal of Optimization Theory and Applications* 64 (1990), 71ff.
- P. P. Wakker, "The Algebraic versus the Topological Approach to Additive Representations", *Journal of Mathematical Psychology*, 32 (1988), 421ff.
- P. P. Wakker, "Transforming Probabilities Without Violating stochastic Dominance", in E. E. Roskam (Ed.), *Mathematical Psychology in Progress*, 1989. Berlin: Springer Verlag.
- P. P. Wakker, "Continuous Subjective Expected Utility With Non-Additive Probabilities", *Journal of Mathematical Economics*, 18 (1989), 1ff.
- Ralph L. Keeney, "Mortality Risks Induced by Economic Expenditures", *Risk Analysis* 10, 147ff.
- Editor's Note:** Because expensive safety regulations have to be financed in some fashion, and because 'poorer' is likely to be 'riskier', programs intended to *save* lives may turn out to *cost* lives instead. This paper furnishes an analytical framework for examining this important problem.